

Distilling Parallel Gradients for Fast ODE Solvers of Diffusion Models

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1. Background

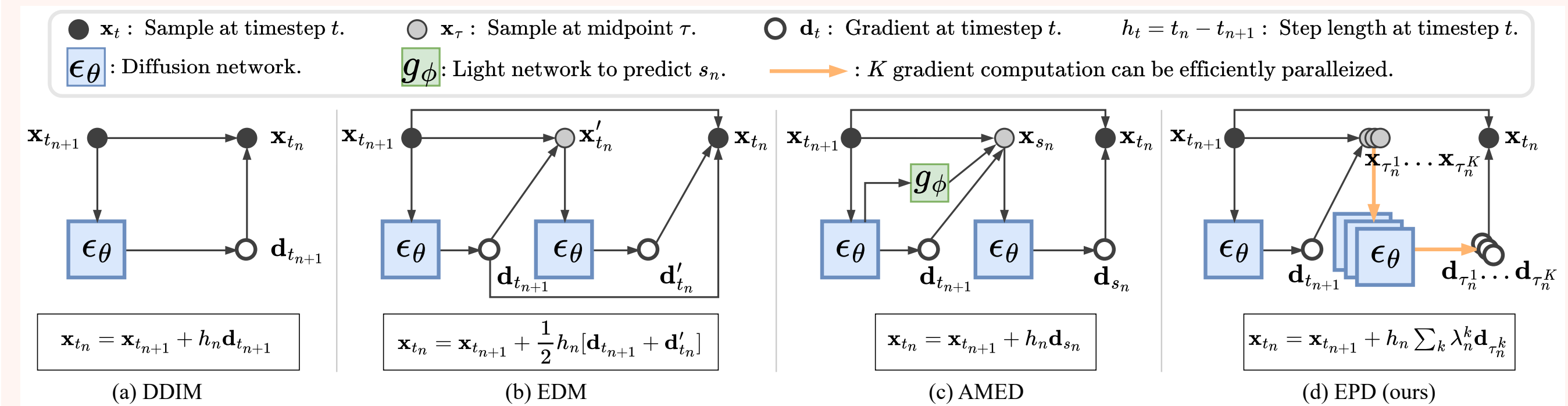
Diffusion models (DMs) have become a leading paradigm in generative modeling. DMs operate by gradually refining a noisy input through a denoising process, producing high-fidelity outputs. However, the multi-step sequential denoising process **introduces substantial latency**. Recent efforts on accelerating DMs typically fall into three categories.

- **Solver-based methods** develop fast numerical solvers to reduce sampling steps. However, inherent truncation errors lead to significant quality degradation when the number of function evaluations (NFE) is low (e.g., < 5).
- **Distillation-based methods** distill student DMs to generate high-quality samples within a minimal number of NFEs, often as low as one. However, this requires extensive training with carefully designed objectives, making the distillation process computationally expensive.
- **Parallelism-based methods** accelerate diffusion models by trading computation for speed. While promising, this direction under low NFEs remains underexplored.

2. Motivation

At a high level, various existing ODE solvers utilize gradients at different timesteps to approximate the ODE solution with varying accuracy. As shown in Figure below:

- DDIM [5] adopts the rectangle rule that uses the gradient at the start point $\mathbf{d}_{t_{n+1}}$.
- EDM [1] adopts the trapezoidal rule that averages the gradients of the start point $\mathbf{d}_{t_{n+1}}$ and the end point \mathbf{d}'_{t_n} .
- AMED [6] optimizes a small network g_ϕ to output an intermediate timestep $s_n \in (t_n, t_{n+1})$ to compute the gradient \mathbf{d}_{s_n} .



Compared to DDIM, EDM and AMED introduce an **additional timestep** for gradient computation (t_n and s_n), leading to improved integral estimation. The key motivation behind our EPD-Solver is to further leverage **multiple timesteps**:

- Compute the gradients at K intermediate timesteps **in parallel** $\mathbf{d}_{\tau_n^1}, \dots, \mathbf{d}_{\tau_n^K}, \tau_n^k \in (t_{n+1}, t_n)$.
- Combine the K gradients via **convex aggregation**, yielding a more precise integral approximation ($\sum_k \lambda_n^k \mathbf{d}_{\tau_n^k}$).
- As the computation of gradients are independent, **the latency remains unchanged**.

3. Theoretical Justification

The use of gradients estimated at multiple timesteps for improved integral approximation can be theoretically justified by the following mean value theorem for vector-valued functions [4]:

When f has values in an n -dimensional vector space and is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) , we have

$$f(b) - f(a) = (b - a) \sum_{k=1}^n \lambda_k f'(c_k), \quad (1)$$

for some $c_k \in (a, b)$, $\lambda_k \geq 0$, and $\sum_{k=1}^n \lambda_k = 1$.

In the context of denoising process, the function outputs an d -dimensional vector as $\mathbf{x} \in \mathbb{R}^d$. The exact integral of $\epsilon_\theta(\mathbf{x}_t, t)$ over the interval $[t_n, t_{n+1}]$ can be expressed as a simplex-weighted combination of gradients evaluated at d intermediate points, scaled by the interval length $h_n = t_n - t_{n+1}$, as formulated in our EPD-Solver.

4. Method: EPD-Solver

Definition of parameters and inference

We define the parameters at step n as $\Theta_n = \{\tau_n^k, \lambda_n^k, \delta_n^k, o_n\}_{k=1}^K$:

- $\tau_n^k \in (t_{n+1}, t_n)$: k -th intermediate timestep.
- $\lambda_n^k \geq 0, \sum_k \lambda_n^k = 1$: combining weights.
- δ_n^k : k -th timestep perturbation.
- o_n : scaling perturbation. δ_n^k and o_n are proposed to mitigate exposure bias [2].

Update rule:

$$\mathbf{x}_{t_n} = \mathbf{x}_{t_{n+1}} + (1 + o_n) h_n \sum_{k=1}^K \lambda_n^k \epsilon_\theta(\mathbf{x}_{\tau_n^k}, \tau_n^k + \delta_n^k)$$

Distillation-based optimization

- **Generating accurate teacher trajectory:** Given a student time schedule with N steps $\mathcal{T}_{\text{stu}} = \{t_0 = t_{\min}, \dots, t_N = t_{\max}\}$, we insert M intermediate steps between t_n and t_{n+1} , i.e., $\mathcal{T}_{\text{tea}} = \{t_0, \dots, t_n, t_n^1, \dots, t_n^M, t_{n+1}, \dots, t_N\}$. The teacher trajectories are generated by any ODE solver (e.g. DPM-Solver) and store the reference states as $\{\mathbf{y}_{t_n}\}_{n=0}^N$.
- **Generate student trajectory:** We sample student trajectory $\{\mathbf{x}_{t_n}\}_{n=0}^N$ with the same initial noise \mathbf{y}_{t_N} , and use the update rule with the parameters $\{\Theta_n\}_{n=1}^N$.
- **Alignment:** We align the two trajectories w.r.t some distance measurement $\text{dist}(\cdot, \cdot)$: $\mathcal{L}_n(\Theta_{1:n}) = \text{dist}(\mathbf{x}_{t_n}, \mathbf{y}_{t_n})$.

EPD-Plugin to existing solvers

EPD-Solver augments existing solvers by substituting their gradient estimation with parallel branches. We illustrate this using the multi-step iPNDM [3]; see the main paper for details.

5. Experiments

| | Method | (Para.) NFE | | | |
|-------------|-------------------|--------------|-------------|-------------|-------------|
| | | 3 | 5 | 7 | 9 |
| Single-step | DDIM | 93.36 | 49.66 | 27.93 | 18.43 |
| | EDM | 306.2 | 97.67 | 37.28 | 15.76 |
| | DPM-Solver-2 | 155.7 | 57.30 | 10.20 | 4.98 |
| | AMED-Solver | 18.49 | 7.59 | 4.36 | 3.67 |
| Multi-step | DPM-Solver++(3M) | 110.0 | 24.97 | 6.74 | 3.42 |
| | UniPC | 109.6 | 23.98 | 5.83 | 3.21 |
| | iPNDM | 47.98 | 13.59 | 5.08 | 3.17 |
| | AMED-Plugin | 10.81 | 6.61 | 3.65 | 2.63 |
| Parallel | ParaDiGMS | 51.03 | 18.96 | 7.18 | 6.19 |
| | EPD-Solver (ours) | 10.40 | 4.33 | 2.82 | <u>2.49</u> |
| | EPD-Plugin (ours) | <u>10.54</u> | <u>4.47</u> | <u>3.27</u> | 2.42 |

(a) Unconditional CIFAR10 32 × 32

| | Method | (Para.) NFE | | | |
|-------------|-------------------|--------------|-------------|-------------|-------------|
| | | 3 | 5 | 7 | 9 |
| Single-step | DDIM | 78.21 | 43.93 | 28.86 | 21.01 |
| | EDM | 356.5 | 116.7 | 54.51 | 28.86 |
| | DPM-Solver-2 | 266.0 | 87.10 | 22.59 | 9.26 |
| | AMED-Solver | 47.31 | 14.80 | 8.82 | 6.31 |
| Multi-step | DPM-Solver++(3M) | 86.45 | 22.51 | 8.44 | 4.77 |
| | UniPC | 86.43 | 21.40 | 7.44 | 4.47 |
| | iPNDM | 45.98 | 17.17 | 7.79 | 4.58 |
| | AMED-Plugin | 26.87 | 12.49 | 6.64 | 4.24 |
| Parallel | ParaDiGMS | 43.64 | 20.92 | 16.39 | 8.81 |
| | EPD-Solver (ours) | <u>21.74</u> | 7.84 | 4.81 | <u>3.82</u> |
| | EPD-Plugin (ours) | 19.02 | <u>7.97</u> | <u>5.09</u> | 3.53 |

(b) Unconditional FFHQ 64 × 64

See the main paper for qualitative results.

References

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